JOURNAL OF AIRCRAFT
Vol. 36, No. 1, January–February 1999

Statistical Approximations for Multidisciplinary Design Optimization: The Problem of Size

Patrick N. Koch*
Engineous Software, Inc., Morrisville, North Carolina 27560
and
Timothy W. Simpson,† Janet K. Allen,‡ and Farrokh Mistree§
Georgia Institute of Technology, Atlanta, Georgia 30332-0405

Despite the steady increase of computing power and speed, the complexity of many of today's engineering analysis codes seems to keep pace with computing advances. Furthermore, the design and development of complex systems typically requires the integration of multiple disciplines and the resolution of multiple conflicting objectives. A departure from the traditional parametric design analysis and single-objective optimization approaches is necessary for the effective solution of multidisciplinary, multiobjective complex design problems that rely on computer analyses. Statistical design of experiments and response surface modeling have been used extensively to create inexpensive-to-run approximations of expensive-to-run computer analyses and combat the problem of size associated with large, multidisciplinary design problems. However, these statistical approaches also break down because of the curse of dimensionality, wherein the number of design variables becomes too large to build accurate response surfaces efficiently. Speculations have been offered in the literature regarding the manageable problem size when these approaches are employed. In this paper, the limitations of these approaches are investigated and demonstrated explicitly by pushing the limits in a large-scale design problem. The design of a high-speed civil transport aircraft wing is used to illustrate 1) the use of these statistical techniques to facilitate multidisciplinary design optimization and 2) the resulting curse of dimensionality associated with large variable design problems. Our current research efforts in system partitioning and hierarchical modeling, and kriging (an alternative statistical approximation technique) are discussed as remedies for the problem of size.

Nomenclature

x = design variables over which the designer has control
y = system responses that represent objectives and constraints
$\hat{y}$ = predicted value of response y at untried value of x
z = noise factors beyond the control of designers
μ_y = mean of a response
σ_y^2 = variance of a response

I. Introduction

MUCH of today's engineering analysis work consists of running complex computer analyses, supplying a vector of design variables (inputs) $\boldsymbol{x}$, and receiving a vector of responses (outputs) $\boldsymbol{y}$. Despite the steady and continuing growth of computing power and speed, many of these analyses remain computationally expensive to perform. Furthermore, the design and development of complex systems typically requires the integration of multiple disciplines and the resolution of multiple conflicting objectives. To facilitate the effective solution of multidisciplinary, multiobjective design problems, a departure from the traditional parametric design analysis and single-objective optimization approaches is needed for the early stages of design.

The inadequacies of traditional parametric design approaches when applied to large-scale, complex systems can be attributed to the problem of size: approaches sufficient for small, simple problems become inefficient and inappropriate as the size of a problem is increased. There are three specific issues relating to problem size that cause traditional approaches to break down: 1) number of variables and responses, 2) computational expense, and 3) multiple objectives with uncertainty.

1) Number of variables and responses: As the number of variables in a design problem increases, comprehensive parametric analysis becomes very time intensive. Often a one-at-a-time approach is used to vary one variable while holding the others fixed, iterating for each variable. This approach is not only inefficient for a large number of variables, but it does not provide sufficient insight into possible interaction effects between variables. Also, individual subsystem and/or discipline analyses are often performed independently, again without considering interaction effects, which leads to excessive iteration to arrive at compatible solutions. Furthermore, as the number of constraints and objectives increases, identification of optimal or good settings for design variables becomes extremely difficult, and parametric approaches become increasingly inefficient.

2) Computational expense: The analysis of complex, multidiscipline systems such as aircraft commonly requires running complex computer analyses. Despite continual advances in computing power, the complexity of these codes seems to keep pace with computing advances, making comprehensive parametric analysis very time intensive.

Received Nov. 11, 1997; revision received April 27, 1998; accepted for publication April 27, 1998. Copyright © 1998 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Mechanical Engineer, Advanced Technologies and Applications, 1800 Perimeter Park West, Suite 275. Member AIAA.

†NSF Graduate Research Fellow, Systems Realization Laboratory, George W. Woodruff School of Mechanical Engineering; currently Assistant Professor of Mechanical Engineering and Industrial Engineering, Pennsylvania State University, 207 Hammond Building, University Park, PA 16802. Member AIAA.

‡Senior Research Scientist, Systems Realization Laboratory, George W. Woodruff School of Mechanical Engineering. Member AIAA.

§Professor, Systems Realization Laboratory, George W. Woodruff School of Mechanical Engineering. Senior Member AIAA.

3) Multiple objectives with uncertainty: The evaluation of complex system alternatives requires addressing multiple performance and economic requirements. The high level of uncertainty in these requirements in the preliminary stages of design complicates the problem of resolving tradeoffs. Given this problem of multiobjective tradeoffs in the face of uncertainty, a significant shift from optimizing to identifying robust regions of good solutions is needed in the preliminary design stages.

One approach to address the first two of these problems of size is through design of experiments and statistical approximation techniques. Our approach to address the third issue is through robust compromise: the integration of robust design principles and the compromise decision support problem. We review each of these techniques in the next section. In Sec. III we present the design of a high speed civil transport (HSCT) aircraft to illustrate the use of statistical approximations and robust compromise to combat the problem of size. However, with large problems like the HSCT example, this approach also breaks down because of the curse of dimensionality, wherein the number of design variables becomes too large to build accurate response surfaces efficiently. Our current research efforts in 1) system decomposition and multilevel optimization and 2) kriging, an alternative statistical approximation technique for modeling computer experiments, for combating the curse of dimensionality associated with large variable problems, are discussed in Sec. IV. Section V contains our closing comments.

II. Frame of Reference: Statistical Approximation Techniques and Robust Compromise

Statistical techniques are widely used in multidisciplinary design to construct approximations of the analyses that are much more efficient to run, easier to integrate together, and yield insight into the functional relationship between $\boldsymbol{x}$ and $\boldsymbol{y}$. Building approximations of computer analyses typically involves 1) choosing an experimental design to sample the computer analysis code, 2) choosing a model to represent the data, and 3) fitting the model to the observed data. There are a variety of options for each of these steps as shown in Fig. 1, and we have attempted to highlight a few of the more prevalent ones, e.g., response surface methodology, and kriging.

In Ref. 1, we discuss several of the approaches listed in Fig. 1, with emphasis on response surface methodology, neural networks, inductive learning, and kriging, and the limitations associated with each. In this paper, our focus is on the general applicability of response surface models for building approximations of computer analyses to combat the problem of size, as described next.

A. Design of Experiments and Response Surface Modeling

A designer's ultimate goal is most often to arrive at improved or robust solutions: determine the values of design variables that best meet the design objectives. In statistical terms, design variables are factors, and design objectives and constraints are responses; the factors and responses to be investigated for a particular design problem provide the input for the approach of Fig. 2, and the solutions (improved or robust) are the output. To identify these solutions, this approach includes three sequential stages that address the three characteristic problems of size introduced in Sec. I (a one-to-one correspondence exists between the problem and the approach):

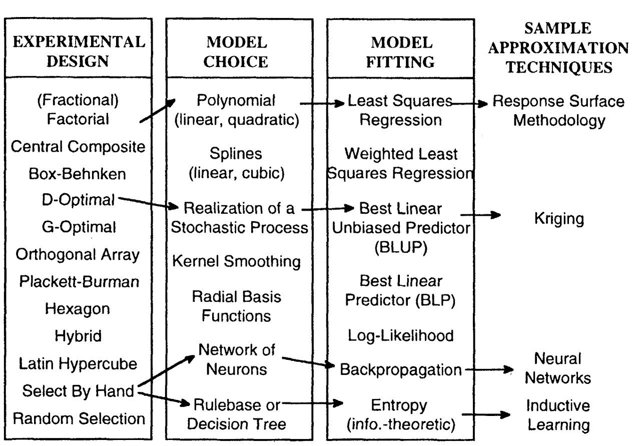

Fig. 1 Techniques for building approximations.[1]

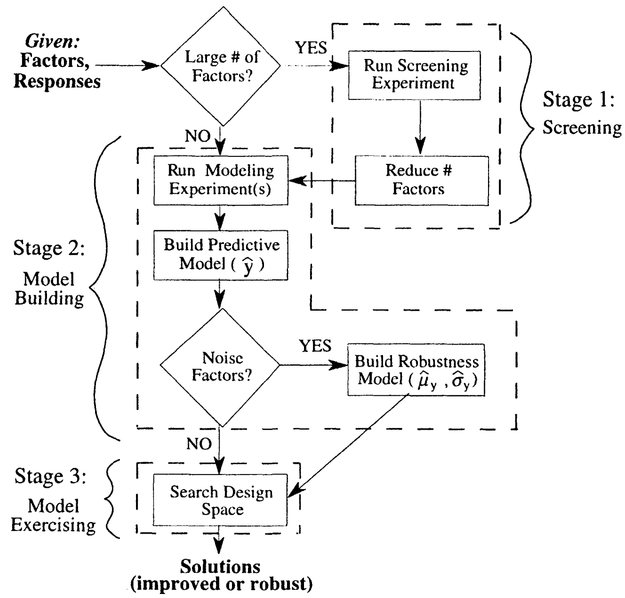

Fig. 2 General approach to response surface modeling.

1) screening → problem of number of variables
2) model building → problem of computational expense
3) model exercising → problem of multiple objectives and uncertainty

The first step (screening) is employed only if the problem includes a large number of factors, e.g., >10; screening experiments are used to reduce the set of factors to those that are most important to the response(s) being investigated. Statistical experimentation is used to define the appropriate design analyses that must be run to evaluate the desired effects of the factors. Often, two level fractional factorial designs or Plackett–Burman designs[2] are used for screening, and only the main (linear) effects of each factor are investigated.

In the second stage (model building) of the approach in Fig. 2, response surface models are created to replace computationally expensive analyses and facilitate fast analysis and exploration of the design space. The most widely used response surface approximating functions are low-order polynomials relating a predicted response, $\hat{\boldsymbol{y}}$, to a set of design variables, $\boldsymbol{x}$. If little curvature exists, a two-level (fractional) factorial experiment is designed, and the first-order polynomial [Eq. (1)], is used to approximate the response(s). If significant curvature exists, a second-order polynomial [Eq. (2)], including all two-factor interactions, is commonly used. Among the various types of experimental designs for fitting a second-order response surface model and studying second-order effects, the central composite design (CCD) is probably the most widely used.[3] CCDs are the first-order fractional factorial designs augmented by an additional star and center points that allow for the creation of a second-order surface with fewer points than would be required for a three-level full factorial design.

The parameters (b terms) of the polynomials in Eqs. (1) and

(2) are usually determined using a least-squares regression analysis to fit the response surface model to observed data:

$$\hat{y} = b_0 + \sum_{i=1}^{k} b_i x_i \tag{1}$$

$$\hat{y} = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i} \sum_{\substack{j \\ i<j}} b_{ij} x_i x_j \tag{2}$$

If noise factors are included in the experimentation, the mean and variance of each response must be estimated, and predictive approximations for both must be built. One approach for building robustness models is presented in the next section.

B. Robust Design

The focus in robust design is to reduce the variation of system performance caused by uncertain design parameters or to reduce system sensitivity. In Fig. 3 we illustrate the concept behind the robust design approach.[4] The control factors, $\boldsymbol{x}$, are adjusted to dampen the variations caused by the noise factors, z. The two curves in Figure 3 represent the performance variation as a function of z when $\boldsymbol{x}$ is at two different levels, $\boldsymbol{x} = a$ and $\boldsymbol{x} = b$. If the design objective is to achieve a performance as close as possible to the target, M, the designs at both levels of $\boldsymbol{x}$ are acceptable; their means are at the target M. However, introducing robustness, when $\boldsymbol{x} = a$, the performance varies significantly with the variation of z. When $\boldsymbol{x} = b$, the performance varies much less; therefore, $\boldsymbol{x} = b$ is better than $\boldsymbol{x} = a$ as a robust design solution because it achieves the mean target and dampens the effect of the noise variation on the response.

To implement robust design, equations for both mean and variance of each response as a function of the control variables are needed. One approach for estimating or modeling the response variation caused by noise is to approximate statistically the response mean and variance. In the model-building stage of the approach in Fig. 2, the modeling experiment designed should include both control and noise factors. A resulting response surface model can then be postulated as a single, formal model of the type:

$$\hat{y} = f(\boldsymbol{x}, z) \tag{3}$$

where $\hat{y}$ is the estimated response, and $\boldsymbol{x}$ and z represent the settings of the control and noise variables, respectively. Based on the response surface model, it is possible to estimate the mean using the statistical expected value function and to estimate the variability of the response using the Taylor series expansion.[5]

Mean of the response:

$$\mu_{\hat{y}} = f(\boldsymbol{x}, \mu_z) \tag{4}$$

Variance of the response:

$$\sigma_{\hat{y}}^2 = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial z_i}\right)^2 \sigma_{z_i}^2 \tag{5}$$

where μ represents the mean values, k is the number of noise factors in the response surface model, and $\sigma_{z_i}^2$ is the variance associated with each noise factor.

The underlying assumptions for this approach are that all noise factors follow a normal distribution, and that Eqs. (4) and (5) provide sufficient estimates of the response mean and variance, respectively. The mean of each noise factor, μ_z, is approximated using the midpoint of the range over which that factor is expected to vary, and the variance is determined by defining the expected range of each noise factor as $\pm 3\sigma$. Given these approximations, and the second-order polynomial response surface models in $\boldsymbol{x}$ and z, the approximations for mean and variance for each response can be derived directly and incorporated within a compromise decision support problem (DSP) to evaluate tradeoffs associated with achieving performance targets and minimizing performance variation. The compromise DSP is described in the next section.

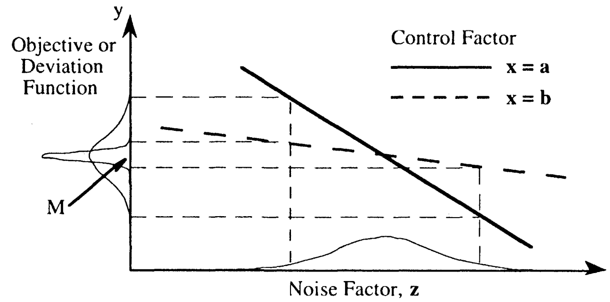

Fig. 3 Robust design approach.[4]

C. Compromise Decision Support Problem

In the third stage of the approach in Fig. 2, the response surface models created in Stage 2 are exercised to explore the design space efficiently (polynomial models are essentially free computationally) and comprehensively (investigate multiobjective tradeoffs and robust design issues). We facilitate this stage through robust compromise: the incorporation of response surface models and robust design principles within the compromise DSP.

The compromise DSP is a multiobjective mathematical construct that is a hybrid formulation based on mathematical programming and goal programming[6] (Fig. 4). The compromise DSP is used to determine the values of the design variables that satisfy a set of constraints and achieve, as closely as possible, a set of conflicting goals (design objectives), as is often the case in robust design and complex systems design. The compromise DSP is solved using the adaptive linear programming algorithm, which is based on sequential linear programming and is part of the decision support in designing engineering systems software (DSIDES).

In Fig. 4, each system goal is modeled using two deviation variables (d_i^-, d_i^+), representing underachievement or overachievement of each goal with respect to the target, G_i. Inclusion of the constraint [$d_i^- \cdot d_i^+ = 0$] requires that one of the deviation variables is always 0, so that mathematically a goal is either overachieved or underachieved, but not both. In the compromise DSP, goals are formulated as equalities, setting appropriate targets G_i, to effectively evaluate the tradeoffs between the multiple goals. To handle these tradeoffs, the objective is to minimize the deviation function, Z, a function of all the deviation variables. Goals may either be weighted in an Archimedean solution scheme or rank-ordered into priority levels using a pre-emptive approach to effect a solution on the basis of preference. For the pre-emptive approach, we use the lexicographic minimum concept[7] to quickly evaluate different design scenarios by changing the priority levels of the goals to be achieved; a similar implementation is discussed in Ref. 8. Using these approaches, or a combination of the two, different design scenarios can be easily and quickly evaluated. A more extensive discussion of deviation variables, deviation functions, system constraints, goals, bounds, and the solution algorithm can be found in Ref. 6.

III. HSCT Wing Design Example

The purpose of the robust design approach described in Sec. II is to address the problems of size discussed in Sec. I, and to facilitate efficient *and* comprehensive concept exploration for large-scale, complex systems. However, as the size of design problems continues to increase, the problem of size returns to also affect this robust design approach. Our focus in this section, then, is to apply this approach to the design of a

Given

An alternative to be improved. Assumptions used to model the domain of interest.

System parameters:

- n number of system variables
- $p + q$ number of system constraints (q inequalities)
- m number of system goals
- $g_{i_3}(\mathbf{x})$ system constraint functions
- $f_k(d_{i_2})$ function of deviation variables to be minimized at priority level k for the preemptive case.

Find: x_{i_1} $i_1 = 1, \ldots, n$; d_i^-, d_i^+ $i_2 = 1, \ldots, m$

Satisfy

System constraints (linear, non-linear):

$$g_{i_3}(\mathbf{x}) = 0 \qquad i_3 = 1, \ldots, p$$
$$g_{i_4}(\mathbf{x}) \geq 0 \qquad i_4 = p+1, \ldots, p+q$$

System goals (linear, non-linear):

$$A_{i_2}(\mathbf{x}) + d_{i_2}^- - d_{i_2}^+ = G_{i_2} \qquad i_2 = 1, \ldots, m$$

Bounds

$$x_{i_1}^{min} \leq x_{i_1} \leq x_{i_1}^{max} \qquad i_1 = 1, \ldots, n$$
$$d_{i_2}^-, d_{i_2}^+ \geq 0 \qquad i_2 = 1, \ldots, m$$
$$d_{i_2}^- \bullet d_{i_2}^+ = 0 \qquad i_2 = 1, \ldots, m$$

Minimize: *deviation function*:

$$Z = [\, f_1(d_{i_2}^-, d_{i_2}^+), \ldots, f_k(d_{i_2}^-, d_{i_2}^+) \,]$$

Fig. 4 Mathematical form of a compromise DSP.[6]

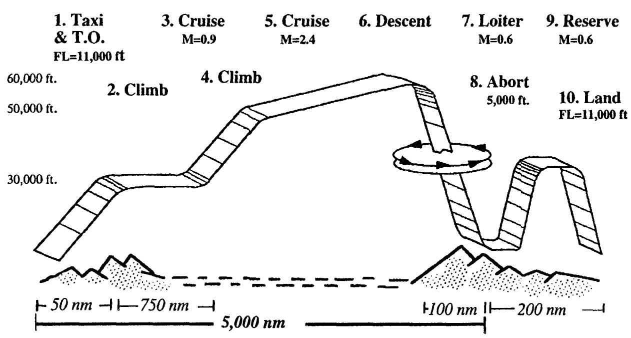

Fig. 5 HSCT mission profile.

Table 1 Objective or goal responses

Response	Target
Takeoff gross weight	750,000 lb
Fuel weight	400,000 lb
Average required yield per revenue passenger mile	$0.12/RPM
Acquisition cost	$230M

Table 2 Constraint responses

Response	Constraint value
Takeoff field length	≤10,500 ft
Landing field length	≤11,000 ft
Approach speed	≤155 kn
Wing fuel volume	≥Required fuel volume

large system and identify the issues of problem size associated with this more efficient approach. As an example, we investigate the wing configuration design of an HSCT aircraft, incorporating life-cycle economic uncertainties to identify economically robust solutions.

A. HSCT Problem Definition

The HSCT is a proposed supersonic, 300-passenger commercial aircraft that is capable of flying 5000 n mile cruise with a 15% cruise segment at Mach 0.9, and the remaining 85% at Mach 2.4 (Fig. 5). A 200 n mile reserve segment is included to provide a safety margin should the aircraft be unable to land at its destination.

The objective in this example is to 1) achieve a technically feasible design while attempting to achieve technical and economic targets, and 2) minimize performance variance caused by economic uncertainty. The NASA synthesis tool FLOPS,[9] with the integrated ENGGEN propulsion analysis code, is employed to size the aircraft and propulsion system to fly the mission given in Fig. 3. The NASA aircraft economic analysis code ALCCA[10] is used to perform economic uncertainty analysis of the system.

The responses investigated in this example include four objectives, or goals, and four technical constraints, listed in Tables 1 and 2, respectively. The first two objective responses in Table 1 are the traditional technical objectives: gross takeoff weight and fuel weight. The economic responses are the required average yield per revenue passenger mile ($/RPM) and aircraft acquisition cost. The $/RPM is a metric that captures the interests of the airline and manufacturer through their respective returns on investment and the interests of passengers through ticket price. The targets for these four goal responses are derived based on experience and from previous HSCT studies conducted at Georgia Tech. The constraint responses listed in Table 2 represent the basic constraints for this aircraft in addition to the mission requirements from Fig. 5.

The factors varied in this study include 22 control factors, or system variables, listed in Table 3, and four economic noise factors, listed in Table 4; the high and low values for each factor are included in each table. The first six control factors listed in Table 3 relate to the HSCT wing planform configuration as shown in Fig. 6. These variables are defined as fractions of the semispan, *b*, also shown in Fig. 6. The focus in this example is on identifying robust wing planform configurations: wing configurations that minimize the effects of economic uncertainty (values of the noise factors in Table 4) on the objectives given in Table 1. The remaining control factors in Table 3 pertain to the geometry of the horizontal and vertical tails, the propulsion cycle, and general characteristics of the aircraft.

Table 3 System (control) variables

Control factor	Unit	Name	Low	High
Kink Y location	Semispan	YKINK	0.44	0.58
LE[a] Kink X location	Semispan	X1	1.54	1.69
LE[a] tip	Semispan	X2	2.1	2.36
TE[a] tip	Semispan	X3	2.4	2.58
TE[a] Kink X location	Semispan	X4	2.19	2.36
TE[a] root	Semispan	X5	2.19	2.5
Wing area	ft^2	SWING	8500	9500
Wing apex	% fuselage	XWING	25	28
Horizontal tail area	ft^2	SHT	400	700
Horizontal tail apex	% fuselage	XHT	82	87.4
Vertical tail area	ft^2	SVT	350	550
Vertical tail apex	% fuselage	XVT	82	86.4
Thrust-to-weight ratio	——	FNWTR	0.28	0.32
Turbine inlet temperature	°R	TIT	3050	3140
Bypass ratio	——	BPR	0.36	0.55
Overall pressure ratio	——	OPR	18	22
Fan inlet Mach number	——	FIMN	0.5	0.7
Fan pressure ratio	——	FPR	3.38	4.2
Nozzle thrust coefficient	——	CFG	0.97	0.99
Engine throttle ratio	——	TTR	1.05	1.15
Suppressor area ratio	——	SAR	1.9	4.7
Takeoff thrust multiplier	——	THFACT	0.85	1

[a]LE is leading edge, and TE is trailing edge.

Table 4 Economic noise factors

Noise factor	Unit	Name	Low	High
Production quantity	#	PRODQ	300	750
Utilization	h/yr	UTIL	3000	5000
Economic range	nm	ECONR	3000	4500
Load factor	%	LOADF	0.55	0.85

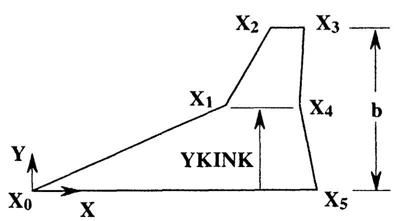

Fig. 6 Wing planform variable definition.

The economic noise factors in Table 4 include the number of aircraft produced (production quantity), the aircraft utilization rate (in hours per year), the economic flight range of the aircraft (in nautical miles), and the average percent of occupied seats (load factor). All of these factors are not under the designer's control but have a significant impact on the economic performance of the aircraft, in this case the $/RPM and acquisition cost. The ranges for these noise factors are determined from predicted demand and knowledge of subsonic routes for which the HSCT may be utilized. In selecting the economic noise factors for this problem, three economic assumptions are made: the airline and manufacturer returns on investment are fixed at 10 and 12%, respectively, and fuel cost is assumed stable at $0.65/gal.

Given the definition of the HSCT problem, the identified mission, responses, factors, and assumptions, the robust design approach of Sec. II can be implemented. It is appropriate before proceeding to apply this approach, however, to ask the question: is the approach of Sec. II warranted for this problem? The answer is clearly yes. For the 26 total factors of this problem, a parametric analysis evaluating only the combinations of the low and high values that define the range of each factor (a two-level full-factorial analysis) would require 67,108,864 executions of FLOPS/ENGGEN and ALCCA. This evaluation does not include any values within the factor ranges, and still is obviously unmanageable (at approximately 5 min per execution on an IBM RISC6000 7012 model 320 Planar workstation, this analysis would take over 600 years to complete). Linking these codes with a gradient-based solver would require on the order of weeks for one complete optimization, and convergence is not guaranteed. With problems of this size, and with complex analysis codes, multiple starting points are essential for design space exploration and solution validation, and the time required again becomes unmanageable. In addition, given the multiple objectives of Table 1 and the incorporation of economic uncertainty, tradeoff compromises and robust design are essential. Thus, the size and characteristics of this problem justify use of the robust design approach in Sec. 2. The screening stage is discussed in the next section.

B. HSCT Screening

For the 26 control and noise factors, a two-level fractional factorial experiment is designed for screening, which consists of 64 experiment points and one center point. Pareto analysis is used to analyze the experimental results to rank the importance of the factors for each response. As an example, Pareto plots for $/RPM and takeoff gross weight are presented in Figs. 7a and 7b, respectively.

In the Pareto plots in Fig. 7, the factors are ranked by relative contribution (individual effect on response), as a fraction of 1, for each response; the bars represent these individual contributions. The curve in each Pareto plot represents the cumulative contribution as each factor contribution is added to the previous sum of contributions. The screening experiment is used to identify the most important factors and reduce the set for higher-order model building. Similar plots, not shown here, are created for the remaining six responses of Tables 1 and 2.

Two important points are taken from the Pareto plots: 1) at least one-third of the factors are required to achieve 80% contribution and over half are needed for 90% contribution, and 2) the ranking of factors is significantly different for each response—different factors are important for different responses. Based on the first point, as the number of factors increases, the number of factors remaining after screening for a single response may still be large (8–10 factors is manageable for creating second-order response surfaces). Given the second point, screening over multiple responses may not allow many factors to be removed from the initial list. These two points are representative of the first characteristic of the problem of size: number of variables and responses.

The standard screening approach for a single response is to select from the factors that have the greatest effect a set of

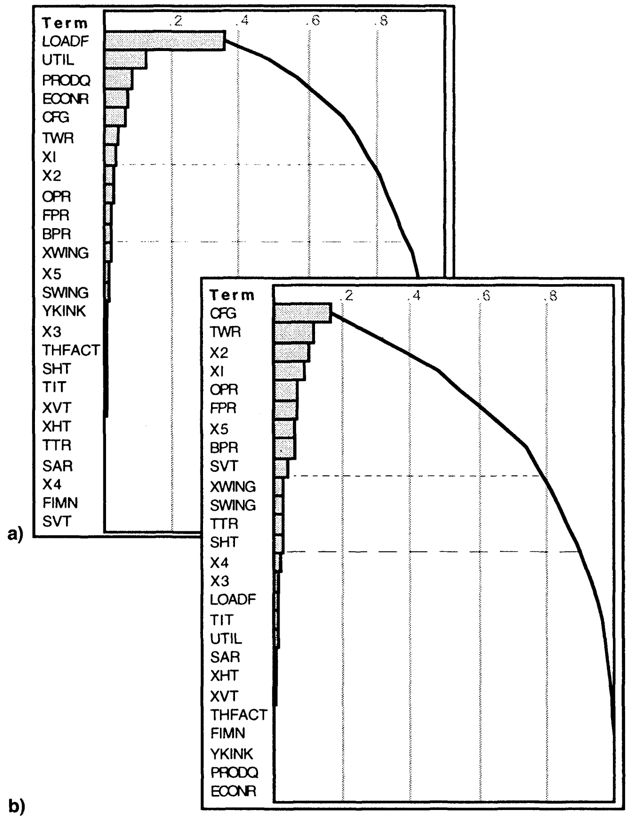

Fig. 7 Pareto plots for a) $/RPM and b) takeoff gross weight.

factors that is small enough to manage, while providing a large enough contribution to the response (80 or 90% contribution, depending on the number of factors, is often used as the cutoff for screening). For the case of multiple responses where each response *likes* different factors, this standard screening approach breaks down. One approach for multiple response screening is to screen each response separately, and select the important factors for each response independently. The model building process is then performed separately for each response: a higher-order experiment is designed and conducted for each response, varying only the factors important to that response. Unfortunately, with this approach, the experimentation time is increased by a factor equal to the number of responses, and the resulting response surface approximations are not consistent—factors varied to create one model are fixed in another. Another approach that has been employed (see Ref. 4) is to average the effects of the factors across all of the responses and select the factors that have the largest effects on average. With this approach it is possible to eliminate a factor that is extremely important for only one response; such a factor may not fair well after averaging.

A reverse screening approach is utilized in this example by identifying factors that are not important for any of the responses. This approach is accomplished by combining the sets of important factors for each response, and observing which factors are not included in the remaining set. An appropriate cutoff point for the acceptable sum of contributions must first be chosen. For this problem, if 90% total contribution is used to identify the important factors for each response (dashed horizontal lines in Fig. 7), the complete set includes all but two of the original factors, leaving a reduced set of 24 factors. If the acceptable total contribution for each response is reduced to 80% (dotted lines in Fig. 7), eight factors are removed, leaving a reduced set of 18 factors. For both the 90 and 80% contribution cutoffs, the number of factors remaining after screening is larger than desirable for fitting a second-order response surface. Reducing the cutoff percentage further, however, leads to a sacrifice in accuracy that is not acceptable (less than 80% of the contribution on the responses is captured; the completeness of problem formulation is less than desirable). [Eighty percent is a somewhat arbitrary cutoff for level of contribution to responses. The tradeoff between an acceptable level of contribution and loss of completeness in problem formulation is very difficult to make. The cutoff point is currently defined by the manageable problem size on a case-specific basis. The focus in this paper to handle this large problem efficiently when approximations are employed through the design of a small composite experiment (see Sec. III.C). Current developments to increase the managable problem size are discussed in Sec. IV.] The tradeoff of accuracy vs efficiency emerges again but now within the approach implemented to address this tradeoff.

Whether either reduced set of factors, 24 or 18, for the HSCT example is larger than manageable is an issue requiring further investigation. To carry this example through robust compromise, the set of 18 factors, the four economic noise factors of Table 4 and the 14 remaining control factors listed in Table 5, is chosen; the problems of size associated with this set are only compounded for the larger set, and the associated level of accuracy is accepted for now.

C. HSCT Modeling Building

Using the remaining 18 control and noise factors, second-order response surface approximations are created for the eight HSCT responses of Tables 1 and 2. At this stage, the second characteristic of the problem of size given in Sec. I (computational expense) arises: a full central composite design in 18 factors would require 262,181 analysis cases, which would take about 2.5 years to complete. Thus, a nonstandard small composite experiment is designed based on a 1/8 fraction of a 2^{11} factorial design (see Ref. 11), which requires 293 runs (about 1.5 times the number of coefficients for a full second-order model in 18 factors). This experiment is nearly three orders of magnitude smaller than a full CCD for 18 factors, and thus, allows the desired efficiency of analysis (1 day rather than 2.5 years to complete the runs). Although the fit of each response led to R^2 values greater than 0.98, the accuracy of predictive models built from small experiments is questionable and is to be tested in the next section.

Mean and variance approximations for robust design are derived from the original response surface models (function of both control and noise factors), for the objectives of Table 1 using the approach given in Sec. II.B [Eqs. (4) and (5)]. These response models are employed for the third stage (model exercising) of the approach in Fig. 2.

D. HSCT Wing Robust Compromise

In this section, we apply our approach for addressing the third characteristic of the problem of size: multiple objectives with uncertainty. The compromise DSP for the HSCT problem is given as follows:

Given:

Mission requirements and planform definition
  300 passengers
  Mach 2.4 cruise
  Range 5000 n mile
  Cruise 15% subsonic, 85% supersonic
Control factors and their ranges (Table 3)
Economic noise factors and their ranges (Table 4)
Response surface models for mean and variance of goal responses in Table 1
Response surface models for constraints in Table 2
HSCT design requirements [constraint values given in Eqs. (6–9)]; goal targets given in Eqs. (10–17)

Find:

The values of the control factors (design variables, Table 3)
The values of the deviation variables d_i^-, d_i^+ [$i = 1, \ldots, 8$; corresponding to goals, Eqs. (10–17)]

Satisfy:

The constraints:

Takeoff field length ≤ 10,500 ft (6)

Landing field length ≤ 11,000 ft (7)

Approach speed ≤ 155 knots (8)

Available wing fuel volume/required fuel volume ≥ 1 (9)

The goals:

$$\mu_{\text{TOGW}}/750{,}000 + d_1^- - d_1^+ = 1.0 \quad (10)$$

$$\sigma^2_{\text{TOGW}}/800{,}000 + d_2^- - d_2^+ = 1.0 \quad (11)$$

$$\mu_{\text{FUELWT}}/400{,}000 + d_3^- - d_3^+ = 1.0 \quad (12)$$

$$\sigma^2_{\text{FUELWT}}/500{,}000 + d_4^- - d_4^+ = 1.0 \quad (13)$$

$$\mu_{\text{\$/RPM}}/0.12 + d_5^- - d_5^+ = 1.0 \quad (14)$$

$$\sigma^2_{\text{\$/RPM}}/0.0001 + d_6^- - d_6^+ = 1.0 \quad (15)$$

$$\mu_{\text{ACQCST}}/230 + d_7^- - d_7^+ = 1.0 \quad (16)$$

$$\sigma^2_{\text{ACQCST}}/100 + d_8^- - d_8^+ = 1.0 \quad (17)$$

Bounds on the control factors: Range limits are given in Table 3

$d_i^- \cdot d_i^+ = 0$, with $d_i^-, d_i^+ \geq 0$

Minimize:

The deviation function: $Z = f(d_i^-, d_i^+)$, where $i = 1-8$ (see HSCT design scenarios, Table 5)

(See Fig. 6 for planform variables.) The information in this formulation is taken from the problem definition in Sec. III.A, and the screening results from Sec. III.B.

To handle the multiple objectives simultaneously, each objective (μ_{TOGW}, σ^2_{TOGW}, etc.) (where TOGW is the takeoff gross weight) is modeled as a goal. Four design scenarios (different deviation functions) are evaluated for the HSCT as listed in Table 5. In the deviation functions of this table, no underachievement deviation variables (d_i^-) are included because for each goal the objective is to minimize only the overachievement. Scenario 1 represents a baseline case for comparison and does not include robustness (only goals for mean response are included). The Archimedean formulation is used with equal

Table 5 HSCT concept exploration scenarios

Scenarios	Deviation function: Priority level 1	Deviation function: Priority level 2
1. Equal weights, no robustness	$0.25*d_1^+ + 0.25*d_3^+ + 0.25*d_5^+ + 0.25*d_7^+$	——
2. Equal weights, with robustness	$0.25*(d_1^+ + d_2^+) + 0.25*(d_3^+ + d_4^+) + 0.25*(d_5^+ + d_6^+) + 0.25*(d_7^+ + d_8^+)$	——
3. Mean on target, then minimum variance	$0.25*d_1^+ + 0.25*d_3^+ + 0.25*d_5^+ + 0.25*d_7^+$	$0.25*d_2^+ + 0.25*d_4^+ + 0.25*d_6^+ + 0.25d_8^+$
4. Minimum variance, then mean on target	$0.25*d_2^+ + 0.25*d_4^+ + 0.25*d_6^+ + 0.25d_8^+$	$0.25*d_1^+ + 0.25*d_3^+ + 0.25*d_5^+ + 0.25*d_7^+$

$*$ indicates multiplication.

Table 6 HSCT robust compromise results[a]

	Design scenarios			
	S1	S2	S3	S4
Control factor				
Kink Y location (YKINK)	0.580	0.561	0.580	0.450
LE[b] Kink X location (X1)	1.540	1.540	1.540	1.619
LE[b] tip (X2)	2.298	2.277	2.108	2.178
TE[b] Kink X location (X4)	2.193	2.190	2.193	2.190
TE[b] root (X5)	2.386	2.416	2.483	2.500
Nozzle thrust coefficient	0.990	0.990	0.990	0.988
Wing area	8500.00	8571.65	8531.00	8562.60
Bypass ratio	0.550	0.508	0.550	0.550
Overall pressure ratio	20.25	19.39	21.91	22.00
Fan pressure ratio	3.508	3.800	3.380	4.052
Takeoff thrust multiplier	0.974	0.966	0.962	0.975
Wing apex	26.30	26.90	25.17	25.28
Vertical tail area	350.00	377.14	540.15	352.67
Thrust-to-weight ratio	0.280	0.281	0.280	0.288
Constraints				
Takeoff field length (ft)	10,210.65	10,487.63	10,475.12	10,509.63
Landing field length (ft)	8411.68	8523.37	8148.42	8710.98
Approach speed (kn)	142.44	143.93	139.96	145.45
Available fuel volume/required fuel volume	1.413	1.306	1.410	1.072
Goals				
Mean takeoff gross weight (lb)	749,960	791,526	**737,848**	860,355
Variance takeoff gross weight	——	556,522	1.11E+7	**491,873**
Mean fuel weight (lb)	400,405	434,082	**396,442**	490,333
Variance fuel weight	——	932,944	9.16E+6	**491,499**
Mean $/RPM ($)	0.125	0.128	**0.126**	0.132
Variance $/RPM	——	8.42E−5	7.51E−5	**8.34E−5**
Mean acquisition cost ($Mil)	264.10	264.35	**263.96**	264.60
Variance acquisition cost	——	164.94	160.34	**159.50**
Deviation function				
Priority level 1	0.0476	0.446	0.0490	0.548
Priority level 2	——	——	8.186	0.156

[a]See Fig. 6 for planform variables.
[b]LE is leading edge and TE is trailing edge.

weights on all four objectives. Scenario 2, representing the overall case including robustness, is for comparison with scenario 1. In scenarios 3 and 4, the pre-emptive formulation is used (with two priority levels), and the Archimedean formulation is used within each priority level. For scenario 3, the goals for the mean of each response are put in the first priority level, representing a focus on trying to achieve mean on target. For the second priority level, the variance goals are included, thus, minimizing variance is addressed only after the mean on target goals are achieved. For scenario 4, the priority levels of scenario 3 are reversed.

Results for the four design scenarios (S1–S4) are given in Table 6. In the first scenario, only one of the four goal targets is achieved (takeoff gross weight). This solution represents the best possible tradeoff between each goal and serves as our baseline design.

Introducing robustness through the variance goals using the formulation of scenario 2, the solution changes slightly. The values achieved for mean gross weight, mean fuel weight, mean \$/RPM, and mean acquisition cost each increase slightly; however, the variances are reduced from the computed variances corresponding to the solution from scenario 1. The results for scenario 2 represent the tradeoff in attempting to simultaneously bring mean on target and minimize variance; system performance is sacrificed to achieve a more robust system. Again, most of the goal targets are not met (only the variance targets of gross weight and \$/RPM are met), and the solution represents a tradeoff or robust compromise.

For scenario 3, the four goals for mean on target are improved significantly. Both the gross weight and fuel weight goals are achieved, and although the \$/RPM solution is not as good as that of scenario 1, it is better than that of scenario 2, as expected. In this case, the variances, placed in the second priority level, are sacrificed to achieve better mean on target; specifically, the gross weight and fuel weight variances increase significantly. The \$/RPM and acquisition cost variances actually improve slightly for this solution, but the total deviation for the variance goals is still higher for scenario 3 than for scenario 2.

For scenario 4, the variances, placed in the first priority level, are reduced substantially; the targets for gross weight and fuel weight variance are both achieved, and the variances of \$/RPM and acquisition cost are reduced compared with the other scenarios. In this case, the values for the mean goals are sacrificed to reduce these variances. The results for this scenario represent the most robust solution.

The corresponding HSCT wing planform configurations are illustrated in Fig. 8. Notice that the resulting configurations for scenarios 1 and 2 (Figs. 8a and 8b, respectively) appear to be very similar. In these cases, the leading- and trailing-edge kink locations (X1 and X4) are basically the same, but the other dimensions are all slightly different. The configuration of scenario 2 represents a more robust planform configuration because the variances in the system objectives caused by economic uncertainty are smaller than the computed variances for scenario 1, which did not include any robust design considerations.

The planform configurations of scenarios 3 and 4 (Figs. 8c and 8d, respectively) are significantly different from those of scenarios 1 and 2, and from each other. For each scenario, the trailing-edge root location (X5) increases. For scenario 3, the leading-edge tip location (X2) moves forward, giving a configuration with a longer chord at the tip. For Scenario 4, the most robust solution, the kink Y location (YKINK) decreases significantly, the leading-edge kink X location (X1) increases, and the trailing-edge root location (X5) increases to its upper bound, giving a configuration with much sharper angles.

E. Summary of HSCT Robust Compromise Results

Our intention in this example is to apply the approach of Sec. II with a focus on identifying additional problems of size

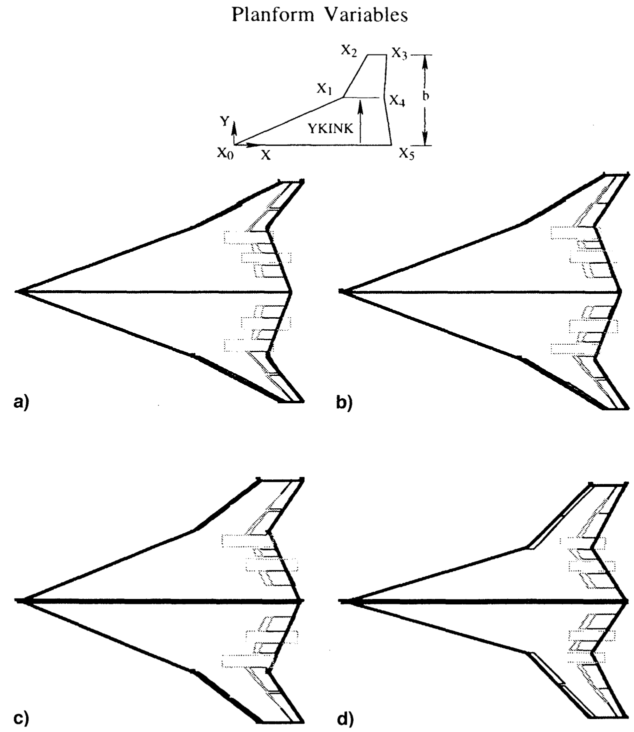

Fig. 8 HSCT wing planform configurations. Scenarios a) 1, b) 2, c) 3, and d) 4.

when implementing this approach. Such issues are discussed in Sec. III.B for screening, and introduced in Sec. III.C for response surface model building. We return now to the discussion of the problem of size associated with model building with a short discussion on verification of results and the accuracy vs efficiency tradeoff.

When employing any approximation models, verification of results is extremely important. The results in Table 6 were analyzed by using the variable values in the original FLOPS/ENGGEN and ALCCA analyses and comparing the results for the eight responses. This comparison is given in Table 7 as a percent error for each response for each design scenario (S1–S4). Many of these errors are higher than desirable, a result that is not surprising given the small number of experiments and the 80% contribution cutoff used for screening. The errors for two of the responses—takeoff field length and the fuel balance constraint—stand out in Table 7. The takeoff field length is underpredicted in all design scenarios, to the extent that each solution is actually infeasible, i.e., the constraint is violated by each configuration. In addition, the fuel balance for scenario 4 is overpredicted to the extent that the available volume for fuel is significantly less than the needed fuel volume; therefore, this configuration would not be able to fly the mission. The errors in gross weight and fuel weight for scenarios 1 and 3 are also higher than desirable.

A tradeoff of accuracy for efficiency occurs in large-scale problems, but as the size of a problem increases, the accuracy can decrease to the point of being unacceptable and the response surface approximations are not useful. The problem of size is not escapable with this approach either because we suffer from the curse of dimensionality resulting from the inefficiency in building accurate second-order response surfaces for problems with a large number of variables. We discuss our current research efforts for handling the problem of size in the next section.

IV. Current Investigations into the Problem of Size

The general question that arises from these problems, then, is how can these experimentation and approximation techniques be used efficiently for larger problems (problems with

Table 7 Verification of results

	% Error[a]			
	S1	S2	S3	S4
Constraints				
Takeoff field length	−7.85	−7.14	−6.68	−5.12
Landing field length	−1.00	−0.065	−2.37	−0.10
Approach speed	−0.67	0.13	−1.39	−0.18
Available fuel/required fuel	24.39	19.92	25.07	15.23
Responses				
Takeoff gross weight	−5.04	−2.53	−5.52	1.14
Fuel weight required	−6.67	−3.20	−6.91	1.85
$/RPM	−1.47	−0.43	−0.47	−0.15
Acquisition cost	−0.19	−0.235	−0.20	−0.13

[a]A negative value indicates value is underpredicted, a positive value indicates value is overpredicted.

greater than 10 factors after screening)? Our current investigations into addressing the problem of size seek to answer the following more specific questions.

1) How can we decompose a large-scale, complex design problem into smaller, more manageable parts?

2) How can decomposed subproblems be evaluated concurrently while ensuring sub-problem compatibility for system synthesis?

3) How can we increase the efficiency of experimentation for larger problems while maintaining sufficient modeling accuracy?

4) How can we efficiently and accurately model highly nonlinear design spaces?

Our research efforts for addressing the first two questions is discussed in Sec. IV.A; a preliminary approach for hierarchical modeling and robust system synthesis is introduced. The third and fourth questions are addressed in Sec. IV.B, wherein kriging is introduced as an alternative statistical approximation technique for modeling large-scale and possibly nonlinear problems.

A. Hierarchical System Partitioning

One technique that may help overcome the problem of size is problem partitioning and multilevel or multidimensional optimization. A recent review of techniques for breaking a problem into smaller problems is given in Ref. 12. Detailed reviews of multidisciplinary design optimization approaches for formulating and concurrently solving decomposed problems are presented in Refs. 13 and 14, and a comparison of some of these approaches is given in Ref. 15.

Hierarchical approaches may be grouped into three categories: 1) single-level optimization, 2) concurrent subspace optimization, and 3) collaborative optimization approaches. Single-level approaches include all-at-once or simultaneous analysis and design (SAND), individual discipline feasible or nested analysis and design, and multidiscipline feasible approaches.[15] In concurrent subspace optimization,[16] the subproblems of a decomposed complex problem are optimized independently and concurrently. A system-level coordinator is then used to evaluate the compatibility of the subproblem solutions (for problems in which system-level design variables and objectives do not exist). In collaborative optimization[17] and multiobjective collaborative optimization,[18] subproblems are optimized concurrently, but a system-level optimizer is used to evaluate compatibility and system-level constraints/objectives, where system-level design variables exist.

In the current literature, these MDO approaches have been applied using complex analysis codes/routines. Very often these analysis codes are computationally expensive and highly nonlinear, leading to difficulties with convergence. We are interested in how the experimentation and approximation techniques presented in this paper can be exploited for these large-scale, complex problems. The approach we are currently investigating integrates problem partitioning and hierarchical modeling techniques with experimentation and approximation

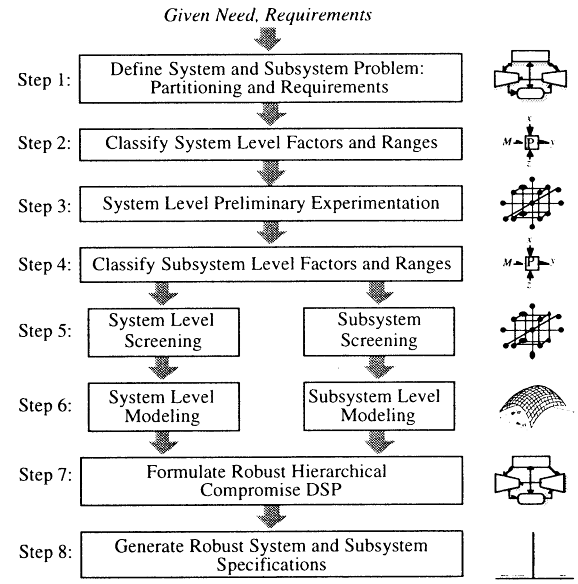

Fig. 9 Approach for hierarchical robust preliminary design exploration.[19]

techniques to facilitate comprehensive and efficient design space exploration for large problems. In particular, we are expanding the robust concept exploration method[4] to support hierarchical robust preliminary design exploration.[19] The focus in this method, shown in Fig. 9, is on facilitating concurrent system and subsystem preliminary design exploration, for concurrent generation of robust system and subsystem specifications for the preliminary design of multilevel, multiobjective, large-scale complex systems.

The method in Fig. 9 includes eight basic steps. Given the need and requirements associated with a preliminary design problem, the problem is partitioned into a system and its subsystems, and the requirements for each are defined (step 1). The system level design factors are then classified using the terminology from robust design, and appropriate factor ranges are defined (step 2). A preliminary system level experimentation (step 3) is used to define necessary subsystem level factor ranges; subsystem factors are then classified and ranges for these factors are defined (step 4). This step is repeated for all subsystems. With both the system and subsystem factors and ranges defined, screening experiments (step 5) are conducted concurrently for the system and all subsystems. Based on the results from screening, higher-order experiments are designed as necessary, and response models are constructed concurrently for the system and subsystem responses (step 6). Using these response models and the problem definition of step 1, a robust, hierarchical compromise DSP is formulated (step 7), and robust system and subsystem specifications are generated concurrently (step 8) for the system and all subsystems.

The method in Fig. 9 represents a hybrid of traditional design and optimization approaches and the more complex multidisciplinary design optimization (MDO) approaches, augmented and facilitated through the development and implementation of response surface approximations. The use of experimentation and approximation within this method allows exploitation of the strengths of traditional design and optimization approaches and more current MDO approaches. After clearly defining the system partitioning (step 1), steps 2–4 follow a traditional top-down sequential design approach. These steps do not require seeking system solutions before investigating subsystem problems; however, they are used to appropriately bound the problem after partitioning, linking the system and subsystem problems. The concurrent screening and model building of steps 5 and 6 are analogous to the distributed analyses typical of all the MDO approaches; the system and subsystem problems are investigated concurrently. The purpose with distributed concurrent analysis, however, is not for optimization or for determining subproblem feasibility (as with the MDO approaches); rather, it is to concurrently identify important system and subsystem factors (step 5), and concurrently construct approximation models (step 6) to replace system and subsystem analyses for efficient and comprehensive exploration of the design space (steps 7 and 8); this can be classified as an all-at-once or the simultaneous analysis and design (SAND) approach. The system and subsystem problems are integrated by formulating a hierarchical compromise DSP. The efficiency gained in using the approximations eliminates the need for distributed/concurrent subsystem evaluation (for feasibility and/or compatibility) or optimization (as with concurrent subspace optimization and collaborative optimization).

The successful implementation of the hierarchical method of Fig. 9 (and the robust design approach of Fig. 2) requires the construction of sufficiently accurate approximation models. Using the HSCT problem of Sec. III as an example, the small composite experiment employed to sample the design space led to models with high prediction errors and the identification of infeasible solutions that were thought to be feasible. Moreover, the primary models we have investigated have been linear and quadratic response surfaces. One aspect of the problem of size that has not been investigated in this paper is the adequacy of these simple models. Quite often the design space associated with a complex system design problem is highly nonlinear and is difficult to reduce to a small region that is good for multiple objectives. The construction of higher-order polynomials, however, increases the computational expense associated with experimentation even further. Thus, alternate approximation techniques are necessary that permit both efficient experimentation and sufficient accuracy in modeling simple and nonlinear design spaces. One such approach that we are investigating is kriging, which is the topic of the next section.

B. Kriging as an Alternative to Response Surface Modeling

To increase the efficiency of experimentation for larger problems while maintaining sufficient modeling accuracy, kriging[20–22] is being investigated as an alternative technique for approximating expensive computer analyses. The kriging approach combines a global underlying model with a locally varying spatial correlation function. The underlying global model is typically taken as a constant but may be linear or quadratic, i.e., Eqs. (1) and (2), if necessary. Meanwhile, there are several possible spatial correlation functions that may be selected for the kriging model[21,22]; the spatial correlation function dictates the local behavior of the model. The Gaussian spatial correlation function that is most commonly used is given by

$$R(\boldsymbol{x}^i, \boldsymbol{x}^j) = \exp\left(-\sum_{k=1}^{\text{numdv}} \theta_k |x_k^i - x_k^j|^2\right) \tag{18}$$

where $\boldsymbol{x}^i$ and $\boldsymbol{x}^j$ are two sample points, numdv is the number of design variables, θ_k is the parameter used to fit the kriging model, and x_k^i and x_k^j are the kth components of $\boldsymbol{x}^i$ and $\boldsymbol{x}^j$. With the Gaussian spatial correlation function, each predicted point is essentially a linear combination of exponentially decaying functions that are based on the spatial distance between x_k^i and x_k^j, thus the name, spatial correlation function.

In our preliminary investigations of the kriging approach, we found that a constant global underlying model and a Gaussian spatial correlation function were as accurate, if not slightly better, than a full second-order response surface model.[23] Kriging models have also been used successfully for several problems involving a large number of variables. Welch et al.[24] describe a kriging-based approximation methodology that they use to identify important variables, detect curvature and interactions, and produce a useful approximation model for two 20-variable problems using only 30–50 runs of the computer code. Booker et al.[25] solve a 31-variable helicopter rotor structural design problem using a similar approximation methodology based on kriging. Booker[26] extends the helicopter rotor design problem to include 56 structural variables to examine the aeroelastic and dynamic response of the rotor.

Why can kriging handle the large number of variables? The response surface approach typically employs second-order polynomials that have $n = (k + 2)(k + 1)/2$ coefficients for k variables, e.g., a second-order response surface for 10 variables has 66 coefficients; for 25 variables, 351 coefficients. Giunta et al.[27] and Kaufman et al.[28] found that $1.5n$ sample points for small problems (5–10 variables) to $4.5n$ sample points for large problems (20–30 variables) are necessary to obtain reasonably accurate second-order response surface approximations. Therefore, the number of samples needed to accurately fit a second-order response surface model grows on the order of k^2, which quickly becomes unmanageable for large variable problems. However, for a kriging model, a constant term, β, is usually sufficient for the global model (as opposed to a second-order polynomial); therefore, the number of design variables is not necessarily dictated by the number of terms needed to fit a polynomial model. The reader is referred to Ref. 23 for more discussion on the differences between the response surface model and kriging approaches.

Finally, there is the issue of accurately modeling highly nonlinear design spaces. As Barton[29] points out, for multicriteria optimization, the response region of interest will rarely be reduced to a small neighborhood by optimization that is necessary to achieve accurate response surfaces. The complications associated with multiple objectives is further evidenced during screening in Sec. III.B; it is difficult to reduce the number of design variables because different variables have different effects on the multiple objectives. Therefore, there is a need to investigate metamodeling techniques with sufficient flexibility to build global approximations of the design space, kriging is one such approximation. A kriging model can either honor the data, providing an exact interpolation of the data, or smooth the data, providing an inexact interpolation.[20] Moreover, a kriging model is extremely flexible because of the wide range of global and local models that can be selected, allowing highly nonlinear design spaces to be modeled with reasonable accuracy, a pertinent issue to MDO that is not considered in this HSCT example.

V. Closing Remarks

In this paper, we identify issues relating to the size of a problem when implementing robust design in the preliminary design stages. We describe an approach for efficient and comprehensive design space exploration for large-scale, complex systems that is founded in robust design principles, statistical experimentation and modeling techniques, and the compromise DSP. We illustrate this approach using the example of an HSCT aircraft with the specific focus of demonstrating the limitations of such approaches; we identify specifically where and when the approach breaks down with problem size as a result of the curse of dimensionality, wherein the number of design variables becomes too large to build accurate response surfaces efficiently.

The first characteristic of the problem of size, as defined in Sec. I, is identified in Sec. III.B for the robust design approach: the screening process breaks down for large numbers of factors and responses. For a single response, as the number of factors increases, the reduction in factors resulting from screening becomes insufficient to bring the problem to a manageable size. So what is a manageable size and at what point does screening break down? If creating a second-order response surface model is the objective, 10 factors or less is generally desirable to build an accurate model with acceptable effort (a full composite design for this case requires 1045 experiments). For most problems, if before screening the number of factors is much greater than 20, screening to 10 or fewer factors usually leads to an unacceptable level of accuracy. This problem with screening is compounded when more than one response is being monitored, as is usually the case with large-scale, complex, multiobjective problems. Often with multiple responses, different factors are important for each response, and as a result, very few factors are not important or are insignificant for all responses. Reducing the number of factors becomes very difficult, as is the case with the HSCT example, and again more factors than desirable, possibly more than manageable, remain after screening.

These problems relating to screening led to the second characteristic of the problem size: computational expense associated with the response model-building stage. If more than 10 factors remain after screening, the computational expense associated with creating response surface approximations can easily begin to outweigh the associated gains, resulting in the curse of dimensionality. Creating second-order response surface models for the HSCT responses in all 26 factors would require 67,108,917 analysis cases (using a full CCD); for the reduced sets of 24 or 18 factors, 16,777,265 and 262,181 cases, respectively, are required, neither of which is manageable. Nonstandard, small composite experiments can be designed with some extra effort as is done for the HSCT example, but a sacrifice in accuracy is associated with fitting models for these smaller experiments.

Our current focus is on addressing these problems of size associated with the robust design approach discussed in this paper, maintaining both efficiency of modeling and exploration, and accuracy necessary for identifying robust regions of a design space. We have implemented and tested the method of Fig. 9 (Sec. IV.A), using an example developed in collaboration with Allison Engine Company, Rolls Royce Aerospace Group: the hierarchical preliminary design of a commercial turbofan engine cycle and compressor subsystem configuration design. This example problem is based on the Allison AE3007 commercial engine developed for regional business jets. The results and the approach are being compared with existing engine data and the traditional engine preliminary design approaches. Favorable results have been obtained and are documented in Ref. 19. As for the kriging approach described in Sec. IV.B, the flexibility of the kriging models and our initial investigations comparing response surfaces and kriging justify continued investigation into the approach despite the added complexity of fitting and using a kriging model.

Acknowledgments

Patrick Koch was supported by a Gwaltney Manufacturing Fellowship, and Timothy Simpson was supported by a National Science Foundation Graduate Research Fellowship. The efforts of the 1996 HSCT Design Team to set up the HSCT example and analysis tools are recognized and greatly appreciated, with special thanks to the team leader, Bryce Roth. We also gratefully thank IMEC of Belgium, particularly Rudi Cartuyvels, for the use of and assistance with the NORMAN software for automated experimental design and analysis. Financial support from National Science Foundation Grants DMI-96-12365 and NASA Grant NGT-51102 is acknowledged. The cost of computer time was underwritten by the Systems Realization Laboratory at the Georgia Institute of Technology.

References

[1]Simpson, T. W., Peplinski, J., Koch, P. N., and Allen, J. K., "On the Use of Statistics in Design and the Implications for Deterministic Computer Experiments," American Society of Mechanical Engineers, Paper DETC97/DTM-3881, Sept. 1997.

[2]Montgomery, D. C., *Design and Analysis of Experiments*, 3rd ed., Wiley, New York, 1991.

[3]Myers, R. H., and Montgomery, D. C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, Wiley, New York, 1995.

[4]Chen, W., Allen, J. K., Mavris, D., and Mistree, F., "A Concept Exploration Method for Determining Robust Top-Level Specifications," *Engineering Optimization*, Vol. 26, No. 2, 1996, pp. 137–158.

[5]Phadke, M. S., *Quality Engineering Using Robust Design*, Prentice–Hall, Englewood Cliffs, NJ, 1989.

[6]Mistree, F., Hughes, O. F., and Bras, B. A., "The Compromise Decision Support Problem and the Adaptive Linear Programming Algorithm," *Structural Optimization: Status and Promise*, edited by M. P. Kamat, AIAA, Washington, DC, 1993, pp. 247–289.

[7]Ignizio, J. P., *Introduction to Linear Goal Programming*, Sage Univ. Papers, Beverly Hills, CA, 1985.

[8]Sandgren, E., "A Multi-Objective Design Tree Approach for Optimization Under Uncertainty," *Advances in Design Automation—Design Optimization*, edited by B. Ravani, DE-Vol. 19-2, American Society of Mechanical Engineers, New York, 1989, pp. 249–255.

[9]McCullers, L. A., *Flight Optimization System, User's Guide, Version 5.7*, NASA Langley Research Center, Hampton, VA, 1993.

[10]Galloway, T. L., and Mavris, D. N., *Aircraft Life Cycle Cost Analysis (ALCCA) Program*, NASA Ames Research Center, Hampton, VA, 1993.

[11]Koch, P. N., Allen, J. K., Mistree, F., and Mavris, D., "The Problem of Size in Robust Design," *Advances in Design Automation*, American Society of Mechanical Engineers, New York, Paper DETC97/DAC-3983, 1997.

[12]Lewis, K., and Mistree, F., "The Other Side of Multidisciplinary Design Optimization: Accomodating a Multiobjective, Uncertain and Non-Deterministic World," *Engineering Optimization* (to be published).

[13]Sobieszczanski-Sobieski, J., and Haftka, R. T., "Multidisciplinary Aerospace Design Optimization: Survey of Recent Developments," AIAA Paper 96-0711, Jan. 1996.

[14]Cramer, E. J., Dennis, J. E., Jr., Frank, P. D., Lewis, R. M., and Shubin, G. R., "Problem Formulation for Multidisciplinary Optimization," *SIAM Journal on Optimization*, Vol. 4, No. 4, 1994, pp. 754–776.

[15]Balling, R. J., and Wilkinson, C. A., "Execution of Multidisciplinary Design Optimization Approaches on Common Test Problems," *Proceedings of the AIAA/NASA/ISSMO 6th Symposium on Multidisciplinary Analysis and Optimization* (Bellevue, WA), AIAA, Reston, VA, 1996, pp. 421–437.

[16]Wujek, B., Renaud, J. E., Batill, S. M., and Brockman, J. B., "Concurrent Subspace Optimization Using Design Variable Sharing in a Distributed Computing Environment," *Concurrent Engineering: Research and Applications (CERA)*, Vol. 4, No. 4, 1996, pp. 361–378.

[17]Sobieski, I. P., and Kroo, I. M., "Aircraft Design Using Collaborative Optimization," AIAA Paper 96-0715, Jan. 1996.

[18]Tappeta, R. V., and Renaud, J. E., "Multiobjective Collaborative Optimization," *Journal of Mechanical Design*, Vol. 119, No. 3, 1997, pp. 403–411.

[19]Koch, P. N., "Hierarchical Modeling and Robust Synthesis for the Preliminary Design of Large Scale, Complex Systems," Ph.D. Dissertation, G. W. Woodruff School of Mechanical Engineering, Georgia Inst. of Technology, Atlanta, GA, 1997.

[20]Cressie, N. A. C., *Statistics for Spatial Data*, Revised, Wiley, New York, 1993.

[21]Koehler, J. R., and Owen, A. B., "Computer Experiments," *Handbook of Statistics*, edited by S. Ghosh and C. R. Rao, Elsevier, New York, 1996, pp. 261–308.

[22]Sacks, J., Welch, W. J., Mitchell, T. J., and Wynn, H. P., "Design and Analysis of Computer Experiments," *Statistical Science*, Vol. 4,

No. 4, 1989, pp. 409–435.

[23]Simpson, T. W., Mauery, T. M., Korte, J. J., and Mistree, F., "Comparison of Response Surface and Kriging Models for Multidisciplinary Design Optimization," *Proceedings of the AIAA/USAF/NASA/ISSMO 7th Symposium on Multidisciplinary Analysis and Optimization* (St. Louis, MO), Vol. 1, AIAA, Reston, VA, 1998, pp. 381–391.

[24]Welch, W. J., Buck, R. J., Sacks, J., Wynn, H. P., Mitchell, T. J., and Morris, M. D., "Screening, Predicting, and Computer Experiments," *Technometrics*, Vol. 34, No. 1, 1992, pp. 15–25.

[25]Booker, A. J., Conn, A. R., Dennis, J. E., Frank, P. D., Serafini, D., Torczon, V., and Trosset, M., "Multi-Level Design Optimization: A Boeing/IBM/Rice Collaborative Project," *1996 Final Report ISSTECH-96-031*, The Boeing Company, Seattle, WA, 1996.

[26]Booker, A. J., "Design and Analysis of Computer Experiments," *Proceedings of the AIAA/USAF/NASA/ISSMO 7th Symposium on Multidisciplinary Analysis and Optimization* (St. Louis, MO), Vol. 1, AIAA, Reston, VA, 1998, pp. 118–128.

[27]Giunta, A. A., Dudley, J. M., Narducci, R., Grossman, B., Haftka, R. T., Mason, W. H., and Watson, L. T., "Noisy Aerodynamic Response and Smooth Approximations in HSCT Design," *Proceedings of the AIAA/USAF/NASA/ISSMO 5th Symposium on Multidisciplinary Analysis and Optimization* (Panama City, FL), Vol. 2, AIAA, Washington, DC, 1994, pp. 1117–1128.

[28]Kaufman, M., Balabanov, V., Burgee, S. L., Giunta, A. A., Grossman, B., Mason, W. H., and Watson, L. T., "Variable-Complexity Response Surface Approximations for Wing Structural Weight in HSCT Design," AIAA Paper 96-0089, Jan. 1996.

[29]Barton, R. R., "Metamodels for Simulation Input-Output Relations," *Proceedings of the 1992 Winter Simulation Conference*, edited by J. J. Swain, D. Goldsman, et al., Inst. of Electrical and Electronics Engineers, New York, 1992, pp. 289–299.

The German V/STOL Fighter Program: A Quest for Survivability in a Theater Nuclear Environment

Albert C. Piccirillo, ANSER

AIAA Case Study
1997, 73 pp, Softcover
ISBN 1-56347-247-3
List Price: $30
AIAA Member Price: $30
Source: 945

This case study examines the German V/STOL fighter and strike aircraft development efforts, the VJ 101 and the VAK 191 programs. The VJ 101C and the VAK 191B design approaches are described, and the results of their flight test efforts are summarized. Designers specializing in V/STOL aircraft design and flight testing will find the lessons of this case study relevant to future ASTOVL fighter and attack aircraft developments.

Call 800/682-AIAA
Order Today!

Visit the AIAA Web site at www.aiaa.org

AIAA
American Institute of Aeronautics and Astronautics
Publications Customer Service
9 Jay Gould Ct.
P.O. Box 753
Waldorf, MD 20604
Phone: 800/682-2422
Fax: 301/843-0159
8 am–5 pm Eastern standard

CA and VA residents add applicable sales tax. For shipping and handling add $4.75 for 1–4 books (call for rates for higher quantities). All individual orders—including U.S., Canadian, and foreign—must be prepaid by personal or company check, traveler's check, international money order, or credit card (VISA, MasterCard, American Express, or Diners Club). All checks must be made payable to AIAA in U.S. dollars, drawn on a U.S. bank. *Orders from libraries, corporations, government agencies, and university and college bookstores must be accompanied* by an authorized purchase order. All other bookstore orders must be prepaid. Please allow 4 weeks for delivery. Prices are subject to change without notice. Returns in sellable condition will be accepted within 30 days. Sorry, we cannot accept returns of case studies, conference proceedings, sale items, or software (unless defective). Non-U.S. residents are responsible for payment of any taxes required by their government.